

Constancy of the Constants of Nature ¹

Takeshi Chiba²

*Department of Physics, Kyoto University,
Kyoto, 606-8502, Japan*

Abstract

The current observational and experimental bounds on time variation of the constants of Nature are briefly reviewed.

1 Introduction

1.1 History

To our knowledge, Dirac appears to have been the first who argued for the possibility of time variation of the constants of nature. As is well-known, dimensionless numbers involving G are huge (or minuscule). For example, the ratio of the electrostatic force to the gravitational force between an electron and a proton is ³

$$N_1 = \frac{e^2}{Gm_pm_e} \simeq 2 \times 10^{39}, \quad (1)$$

where e is the electric charge, m_p is the proton mass and m_e is the electron mass. Similarly, the ratio of the Hubble horizon radius of the Universe, H_0^{-1} to the classical radius of an electron is

$$N_2 = \frac{H_0^{-1}}{e^2 m_e^{-1}} \simeq 3 \times 10^{40} h^{-1}, \quad (2)$$

where h is the Hubble parameter in units of $\text{kms}^{-1}\text{Mpc}^{-1}$. Curiously, the two nearly coincides, which motivated Dirac to postulate the so-called the large number hypothesis [1]. In his article entitled “A new basis for cosmology”, he describes [1]

Any two of the very large dimensionless numbers occurring in Nature are connected by a simple mathematical relation, in which the coefficients are of the order of magnitude unity.

Thus if the (almost) equality $N_1 = \mathcal{O}(1) \times N_2$ holds always, then G must decrease with time $G \propto t^{-1}$ [2], or the fine structure constant, $\alpha \equiv e^2$, must increase with time $\alpha \propto t^{1/2}$ [3] since $H \propto t^{-1}$.

Nowadays we know that such a huge dimensionless number like N_1 is related to the gauge hierarchy problem. In fact, the gauge couplings are *running* (however, only logarithmically) as the energy grows, and all the gauge couplings are believed to unify at

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²E-mail: chiba@tap.scphys.kyoto-u.ac.jp

³We use the units of $\hbar = c = 1$.

the fundamental energy scale (probably string scale). The fact that N_1 nearly coincides with N_2 may be just accidental, and pursuing the relation between them is numerological speculation (or requires anthropic arguments). However, Pandora’s box was opened.

1.2 Modern Motivation

String theory is the most promising approach to unify all the fundamental forces in nature. It is believed that in string theory all the coupling constants and parameters (except the string tension) in nature are derived quantities and are determined by the vacuum expectation values of the dilaton and moduli.

On the other hand, we know that the Universe is expanding. Then it is no wonder to imagine the possibility of time variation of the constants of nature during the evolution of the Universe.

In fact, it is argued that the effective potentials of dilaton or moduli induced by nonperturbative effects may exhibit runaway structure; they asymptote zero for the weak coupling limit where dilaton becomes minus infinity or internal radius becomes infinity and symmetries are restored in the limit [4, 5]. Thus it is expected that as these fields vary, the natural “constants” may change in time and moreover the violation of the weak equivalence principle may be induced [5, 6] (see also [7, 8] for earlier discussion).

In this article, we review the current experimental (laboratory, astrophysical and geophysical) constraints on time variation of the constants of nature. In particular, we consider α , G , and Λ . We assume $H_0 = 100h\text{km/s/Mpc}$ with $h = 0.65$ for the Hubble parameter and $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$ for the cosmological parameters. For earlier expositions, see [9, 10] for example.

2 α

In this section, we review the experimental constraints on time variation of the fine structure constant. The results are summarized in Table 1.

2.1 Oklo Natural Reactor and $\dot{\alpha}$

In 1972, the French CEO (Commissariat à l’Energie Atomique) discovered ancient natural nuclear reactors in the ore body of the Oklo uranium mine in Gabon, West Africa. It is called the Oklo phenomenon. The reactor operated about 1.8 billion years ago corresponding to $z \simeq 0.13$ for the assumed cosmology ($h = 0.65, \Omega_M = 0.3, \Omega_\Lambda = 0.7$).

Shlyakhter noticed the extremely low resonance energy ($E_r = 97.3\text{meV}$) of the reaction

$$^{149}\text{Sm} + n \rightarrow ^{150}\text{Sm} + \gamma, \quad (3)$$

and hence the abundance of ^{149}Sm (one of the nuclear fission products of ^{235}U) observed at the Oklo can be a good probe of the variability of the coupling constants [11]. The isotope ratio of $^{149}\text{Sm}/^{147}\text{Sm}$ is 0.02 rather than 0.9 as in natural samarium due to the neutron flux onto ^{149}Sm during the uranium fission. From an analysis of nuclear and geochemical data, the operating conditions of the reactor was inferred and the thermally

averaged neutron-absorption cross section could be estimated. The nuclear Coulomb energy is of order $V_0 \sim 1\text{MeV}$, and its change ΔV_0 is related to the change of α as

$$\frac{\Delta V_0}{V_0} = \frac{\Delta E_r}{V_0} = \frac{\Delta \alpha}{\alpha}. \quad (4)$$

By estimating the uncertainty in the resonance energy, Shlyakhter obtained $\dot{\alpha}/\alpha = 10^{-17}\text{yr}^{-1}$. Damour and Dyson reanalysed the data and obtained $(-6.7 \sim 5.0) \times 10^{-17}\text{yr}^{-1}$ [12]. Using new samples that were carefully collected to minimize natural contamination and also on a careful temperature estimate of the reactors, Fujii et al. reached a tighter bound⁴ $(-0.2 \pm 0.8) \times 10^{-17}\text{yr}^{-1}$ [13].

2.2 (Hyper)Fine splitting and $\dot{\alpha}$

Since fine structure levels depend multiplicatively on α , wavelength spectra of cosmologically distant quasars provide a natural laboratory for investigating the time variability of α . Narrow lines in quasar spectra are produced by absorption of radiation in intervening clouds of gas, many of which are enriched with heavy elements. For example, the separation between the wavelength corresponding to the transition $^2S_{1/2} \rightarrow ^2P_{3/2}$ and the wavelength corresponding to the transition $^2S_{1/2} \rightarrow ^2P_{1/2}$ in an alkaline ion is proportional to α^2 . Because quasar spectra contain doublet absorption lines at a number of redshifts, it is possible to check for time variation in α simply by looking for changes in the doublet separation of alkaline-type ions with one outer electron as a function of redshift [14, 15]. By looking at Si IV doublet, Cowie and Songaila obtained the limit up to $z \simeq 3$: $|\Delta\alpha/\alpha| < 3.5 \times 10^{-4}$ [16]. Also by comparing the hyperfine 21 cm HI transition with optical atomic transitions in the same cloud at $z \simeq 1.8$, they obtained a bound on the fractional change in α up to redshift $z \simeq 1.8$: $\Delta\alpha/\alpha = (3.5 \pm 5.5) \times 10^{-6}$, corresponding to $\dot{\alpha}/\alpha = (-3.3 \pm 5.2) \times 10^{-16}\text{yr}^{-1}$ [16]. Recently, by comparing the absorption by the HI 21 cm hyperfine transition (at $z = 0.25, 0.68$) with the absorption by molecular rotational transitions, Carilli et al. obtained a bound: $|\Delta\alpha/\alpha| < 1.7 \times 10^{-5}$ [17].

Webb et al. [18] introduced a new technique (called many-multiplet method) that compares the absorption wavelengths of magnesium and iron atoms in the same absorbing cloud, which is far more sensitive than the alkaline-doublet method. They observed a number of intergalactic clouds at redshifts from 0.5 to 1.6. For the entire sample they find $\Delta\alpha/\alpha = (-1.1 \pm 0.4) \times 10^{-5}$, consistent with a null result within 3σ . They noted that the deviation is dominated by measurements at $z > 1$, where $\Delta\alpha/\alpha = (-1.9 \pm 0.5) \times 10^{-5}$.

Recently, Webb et al. [19] presented further evidence for time variation of α by re-analysing the previous data and including new data of Keck/HIRES absorption systems. The results indicate a smaller α in the past and the optical sample shows a 4σ deviation for $0.5 < z < 3.5$: $\Delta\alpha/\alpha = (-0.72 \pm 0.18) \times 10^{-5}$. They noted that the potentially significant systematic effects only make the deviation significant. If the result is correct, it would have profound implications for our understanding of fundamental physics. So the claim needs to be verified independently by other observations.

⁴They noted that data is also consistent with a non-null result: $(-4.9 \pm 0.4) \times 10^{-17}\text{yr}^{-1}$, indicating an apparent evidence for the time variability. However, from the analysis of the isotope compositions of G_d , the consistency of the S_m and G_d results supports the null results.

2.3 Laboratory Tests: Clock Comparison

Laboratory constraints are based on clock comparisons with ultrastable oscillations of different physical makeup such as the superconducting cavity oscillator vs the cesium hyperfine clock transition [20] or the Mg fine structure transition vs the cesium hyperfine clock transition [21]. Since a hyperfine splitting is a function of $Z\alpha$, such a clock comparison can be a probe of time variation of α . The clock comparisons place a limit on present day variation of α and are repeatable and hence are complementary to the geophysical or cosmological constraints. Comparisons of rates between clocks based on hyperfine transitions in alkali atoms with different atomic number Z (H-maser and Hg^+ clocks) over 140 days yield a bound: $|\dot{\alpha}/\alpha| \leq 3.7 \times 10^{-14} \text{yr}^{-1}$ [22].

2.4 Cosmology and $\dot{\alpha}$: Big-Bang Nucleosynthesis and Cosmic Microwave Background

Big-Bang Nucleosynthesis. The ${}^4\text{He}$ abundance is primarily determined by the neutron-to-proton ratio prior to nucleosynthesis which before the freeze-out of the weak interaction rates at a temperature $T_f \sim 1\text{MeV}$, is given approximately by the equilibrium condition:

$$(n/p) \simeq \exp(-Q/T_f), \quad (5)$$

where $Q = 1.29\text{MeV}$ is the mass difference between neutron and proton. Assuming that all neutrons are incorporated into ${}^4\text{He}$, ${}^4\text{He}$ abundance Y_p is given by $Y_p \simeq 2(n/p)/[1 + (n/p)]$. The freeze-out temperature is determined by the competition between the weak interaction rates and the expansion rate of the Universe. The dependence of T_f on the weak and gravitational couplings is given by

$$T_f \propto G_F^{-2/3} G^{1/6}, \quad (6)$$

where G_F is the Fermi constant.

Changes in Y_p are induced by changes in T_f and Q . However, it is found that Y_p is most sensitive to changes in Q [23]. The α dependence of Q can be written as [24, 25, 26]

$$Q \simeq 1.29 - 0.76 \times \Delta\alpha/\alpha \text{ MeV}. \quad (7)$$

Comparing with the observed Y_p , a bound on $\Delta\alpha/\alpha$ is obtained: $|\Delta\alpha/\alpha| \leq 2.6 \times 10^{-2}$ [25]. Recently, it is argued that the presently unclear observational situation concerning the primordial abundances precludes a better bound than $|\Delta\alpha/\alpha| \leq 2 \times 10^{-2}$ [27].

Cosmic Microwave Background. Changing α alters the ionization history of the universe and hence affects the spectrum of cosmic microwave background fluctuations: it changes the Thomson scattering cross section, $\sigma_T = 8\pi\alpha^2/3m_e^2$, and hence the differential optical depth $\dot{\tau}$ of photons due to Thomson scattering through $\dot{\tau} = x_e n_p \sigma_T$ where x_e is the ionization fraction and n_p is the number density of electrons; it also changes the recombination of hydrogen.

The last scattering surface is defined by the peak of the visibility function, $g(z) = e^{-\tau(z)} dt/dz$, which measures the differential probability that a photon last scattered at

	redshift	$\Delta\alpha/\alpha$	$\dot{\alpha}/\alpha(\text{yr}^{-1})$
Atomic Clock[22]	0		$\leq 3.7 \times 10^{-14}$
Oklo(Damour-Dyson[12])	0.13	$(-0.9 \sim 1.2) \times 10^{-7}$	$(-6.7 \sim 5.0) \times 10^{-17}$
Oklo(Fujii et al.[13])	0.13	$(-0.36 \sim 1.44) \times 10^{-8}$	$(-0.2 \pm 0.8) \times 10^{-17}$
HI 21 cm[16]	1.8	$(3.5 \pm 5.5) \times 10^{-6}$	$(-3.3 \pm 5.2) \times 10^{-16}$
HI 21 cm[17]	0.25,0.68	$< 1.7 \times 10^{-5}$	
QSO absorption line[18]	0.5 – 1.6	$(-1.1 \pm 0.4) \times 10^{-5}$	
QSO absorption line[19]	0.5 – 3.5	$(-0.72 \pm 0.18) \times 10^{-5}$	
CMB[28]	10^3	$< 10^{-2} \sim 10^{-3}$	$< 10^{-12} \sim 10^{-13}$
BBN[27]	10^{10}	$< 2 \times 10^{-2}$	$< 1.4 \times 10^{-12}$

Table 1: Summary of the experimental bounds on time variation of the fine structure constant. $\Delta\alpha/\alpha \equiv (\alpha_{\text{then}} - \alpha_{\text{now}})/\alpha_{\text{now}}$. A bound on $\Delta\alpha/\alpha$ for CMB is a possible bound.

redshift z . As explained in [28], increasing α affects the visibility function $g(z)$: it increases the redshift of the last scattering surface and decreases the thickness of the last scattering surface. This is because the increase in α shifts $g(z)$ to higher redshift since the equilibrium ionization fraction, x_e^{EQ} , is shifted to higher redshift and because x_e more closely tracks x_e^{EQ} for larger α .

An increase in α changes the spectrum of CMB fluctuations: the peak positions in the spectrum shift to higher values of ℓ (that is, a smaller angle) and the values of C_ℓ increase [28]. The former effect is due to the increase of the redshift of the last scattering surface, while the latter is due to a larger early integrated Sachs-Wolfe effect because of an earlier recombination. It is concluded that the future of cosmic microwave background experiment (MAP, Planck) could be sensitive to $|\Delta\alpha/\alpha| \sim 10^{-2} - 10^{-3}$ [28, 29].⁵

3 G

In this section, we review the experimental constraints on time variation of the Newton constant. For more detailed review see [33]. The results are summarized in Table 2.

3.1 Viking Radar-Ranging to Mars, Lunar-Laser-Ranging and \dot{G}

If we write the effective gravitational constant G as $G = G_0 + \dot{G}_0(t - t_0)$, the effect of changing G is readily seen through the change in the equation of motion:

$$\frac{d^2 \mathbf{x}}{dt^2} = -\frac{GM\mathbf{x}}{r^3} = -\frac{G_0 M \mathbf{x}}{r^3} - \frac{\dot{G}_0}{G_0} \frac{G_0 M}{r} \frac{\mathbf{x}(t - t_0)}{r^2}. \quad (8)$$

⁵After this talk, new analyses of BOOMERanG [30] and MAXIMA [31] and the first year results from DASI [32] appeared. In those data the second (and even the third) peak is now clearly seen, which discourages the motivation for non-standard recombination scenarios that were attracted attention regarding the interpretation of the apparent absence of the second peak in the previous data.

	redshift	$\Delta G/G$	$\dot{G}/G(\text{yr}^{-1})$
Viking Lander Ranging[34]	0		$(2 \pm 4) \times 10^{-12}$
Lunar Laser Ranging[36]	0		$(1 \pm 8) \times 10^{-12}$
Double Neutron Star Binary[38]	0		$(1.10 \pm 1.07) \times 10^{-11}$
Pulsar-White Dwarf Binary[39]	0		$(-9 \pm 18) \times 10^{-12}$
Helioseismology[43]	0		$< 1.6 \times 10^{-12}$
Neutron Star Mass[44]	$0 - 3 \sim 4$		$(-0.6 \pm 2.0) \times 10^{-12}$
BBN[46]	10^{10}	$-0.3 \sim 0.4$	$(-2.7 \sim 2.1) \times 10^{-11}$

Table 2: Summary of the experimental bounds on time variation of the gravitational constant. $\Delta G/G \equiv (G_{\text{then}} - G_{\text{now}})/G_{\text{now}}$.

Thus time variation of G induces an acceleration term in addition to the usual Newtonian and relativistic ones, which would affect the motion of bodies, such as planets and binary pulsar.

A relative distance between the Earth and Mars was accurately measured by taking thousands of range measurements between tracking stations of the Deep Space Network and Viking landers on Mars. From a least-squares fit of the parameters of the solar system model to the data taken from various range measurements including those by Viking landers to Mars (from July 1976 to July 1982), a bound on \dot{G} is obtained: $\dot{G}/G = (2 \pm 4) \times 10^{-12}\text{yr}^{-1}$ [34].

Similarly, Lunar-Laser-Ranging measurements have been used to accurately determine parameters of the solar system, in particular the Earth-Moon separation. From the analysis of the data from 1969 to 1990, a bound is obtained: $\dot{G}/G = (0.1 \pm 10.4) \times 10^{-12}\text{yr}^{-1}$ [35]; while from the data from 1970 to 1994, $\dot{G}/G = (1 \pm 8) \times 10^{-12}\text{yr}^{-1}$ [36].

3.2 Binary Pulsar and \dot{G}

The timing of the orbital dynamics of binary pulsars provides a new test of time variation of G . To the Newtonian order, the orbital period of a two-body system is given by

$$P_b = 2\pi \left(\frac{a^3}{Gm} \right)^{1/2} = \frac{2\pi\ell^3}{G^2 m^2 (1 - e^2)^{3/2}}, \quad (9)$$

where a is the semi-major axis, $\ell = r^2\dot{\phi}$ is the angular momentum per unit mass, m is a Newtonian-order mass parameter, and e is the orbital eccentricity. This yields the orbital-period evolution rate

$$\frac{\dot{P}_b}{P_b} = -2\frac{\dot{G}}{G} + 3\frac{\dot{\ell}}{\ell} - 2\frac{\dot{m}}{m}. \quad (10)$$

Damour, Gibbons and Taylor showed that the appropriate phenomenological limit on \dot{G} is obtained by

$$\frac{\dot{G}}{G} = -\frac{\delta\dot{P}_b}{2P_b}, \quad (11)$$

where $\delta\dot{P}_b$ represents whatever part of the observed orbital period derivative that is not otherwise explained [37]. From the timing of the binary pulsar PSR 1913+16, a bound is obtained: $\dot{G}/G = (1.0 \pm 2.3) \times 10^{-11} \text{yr}^{-1}$ [37] (see also [38] and [39]). However, only for the orbits of bodies which have negligible gravitational self-energies, the simplifications can be made that \dot{P}_b/P_b is dominated by $-2\dot{G}/G$ term. When the effect of the variation in the gravitational binding energy induced by a change in G is taken into account, the above bound is somewhat weakened depending on the equation of state [40].

3.3 Stars and \dot{G}

Since gravity plays an important role in the structure and evolution of a star, a star can be a good probe of time variation of G . It can be shown from a simple dimensional analysis that the luminosity of a star is proportional to G^7 [41]. Increasing G is effectively the same, via the Poisson equation, as increasing the mass or average density of a star, which increases its average mean molecular weight and thus increases the luminosity of a star. Since a more luminous star burns more hydrogen, the depth of convection zone is affected which is determined directly from observations of solar p -mode (acoustic wave) spectra [42]. Helioseismology enables us probe the structure of the solar interior. Comparing the p -mode oscillation spectra of varying- G solar models with the solar p -mode frequency observations, a tighter bound on \dot{G} is obtained: $|\dot{G}/G| \leq 1.6 \times 10^{-12} \text{yr}^{-1}$ [43].

The balance between the Fermi degeneracy pressure of a cold electron gas and the gravitational force determines the famous Chandrasekhar mass

$$M_{Ch} \simeq G^{-3/2} m_p^{-2}, \quad (12)$$

where m_p is the proton mass. Since M_{Ch} sets the mass scale for the late evolutionary stage of massive stars, including the formation of neutron stars in core collapse of supernovae, it is expected that the average neutron mass is given by the Chandrasekhar mass. Measurements of neutron star masses and ages over $z < 3 \sim 4$ yield a bound, $\dot{G}/G = (-0.6 \pm 2.0) \times 10^{-12} \text{yr}^{-1}$ [44].

3.4 Cosmology and \dot{G} : Big-Bang Nucleosynthesis

The effect of changing G on the primordial light abundances (especially ${}^4\text{He}$) is already seen in Eq.(5) and Eq.(6): an increase in G increases the expansion rate of the universe, which shifts the freeze-out to an earlier epoch and results in a higher abundance of ${}^4\text{He}$. In terms of the “speed-up factor”, $\xi \equiv H/H_{SBBN}$, Y_p is well fitted by [45]

$$Y_p \simeq 0.244 + 0.074(\xi^2 - 1). \quad (13)$$

If Y_p was between 0.22 and 0.25, then $-0.32 < \Delta G/G < 0.08$, which corresponds to $\dot{G}/G = (-0.55 \sim 2.2) \times 10^{-11} \text{yr}^{-1}$. A similar (more conservative) bound was obtained in [46]: $-0.3 < \Delta G/G < 0.4$.

3.5 Recent Developments on G_0

No laboratory measurements of \dot{G}/G has been performed recently (see [33] for older laboratory experiments). This is mainly because the measurements of the present gravitational constant G_0 itself suffer from systematic uncertainties and have not been performed with good precision.

Recently, Gundlach and Merkowitz measured G_0 with a torsion-balance experiment in which string-twisting bias was carefully eliminated [47]. The result was a value of $G_0 = (6.674215 \pm 0.000092) \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$.⁶ As the accuracy of the measurements improves, it may be possible to place a bound on a present-day variation of G . It is important to pursue laboratory measurements of \dot{G}/G since they are repeatable and hence are complementary to astrophysical and geophysical constraints.

4 Λ or Dark Energy

Finally, we briefly comment on the potential variability of the cosmological constant (or dark energy) because in the runaway scenario of dilaton or moduli ϕ , $\dot{\alpha}/\alpha$ and \dot{G}/G would close to $\dot{\phi}/\phi$ [5].

4.1 Evidence for $\Lambda > 0$

There are two arguments for the presence of dark energy. The first indirect evidence comes from the sum rule in cosmology:

$$\sum \Omega_i = 1, \quad (14)$$

where $\Omega_i \equiv 8\pi G\rho_i/3H_0^2$ is the density parameter of the i -th energy component, ρ_i . The density parameter of the curvature, Ω_K , is defined by $\Omega_K \equiv -k/a^2 H_0^2$. Since the current observational data indicate that matter density is much less than the critical density $\Omega_M < 1$ and that the Universe is flat, we are led to conclude that the Universe is dominated by dark energy, $\Omega_{DE} = 1 - \Omega_M - \Omega_K > 0$.

The second evidence for dark energy is from the observational evidence for the accelerating universe [49]:

$$\frac{\ddot{a}}{aH_0^2} = -\frac{1}{2} \left(\Omega_M(1+z)^3 + (1+3w)\Omega_{DE}(1+z)^{3(1+w)} \right) > 0, \quad (15)$$

where w is the equation of state of dark energy, $w \equiv p_{DE}/\rho_{DE}$. Since distance measurements to SNIa strongly indicate the Universe is currently accelerating, the Universe should be dominated by dark energy with negative pressure ($w < 0$). We note that another argument for negative pressure comes from the necessity of the epoch of the matter domination.

⁶Only recently, the measurement of G with a torsion-strip balance resulted in $G_0 = (6.67559 \pm 0.00027) \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$, which is 2 parts in 10^4 higher than the result of Gundlach and Merkowitz [48]. Probably the difference is still due to systematic errors hidden in one or both of the measurements.

4.2 Supernova and $\dot{\Lambda}$

A current bound on the equation of state of dark energy from supernova data is $w \leq -0.6$ [50]. Future observations of high redshift supernovae/galaxies/clusters would pin down the bound on w to $w \leq -0.9$. The extent of time variation of dark energy density is readily seen from the equation of motion:

$$\frac{\dot{\rho}_{DE}}{\rho_{DE}} = -3(1+w)H. \quad (16)$$

5 Conclusion

A short account of the experimental constraints on the time variability of the constants of nature (α and G) was given. Since there are some theoretical motivations for the time variability of the constants of nature and the implications of it are profound, refining these bounds remains important, and continuing searches for the possible time variability of the constants of nature should be made.

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